LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **STATISTICS**

SECOND SEMESTER – **APRIL 2023**

PST 2503 – SAMPLING THEORY

Date: 03-05-2023 Dept. No. Max. : 100 Marks

Time: 01:00 PM - 04:00 PM

SECTION - A

Answer ALL questions. Each carries TWO marks. (10 x 2 = 20 marks)

1. Give an example for a statistic which is unbiased with respect to a sampling design.
2. State and prove a necessary and sufficient condition for the existence of an unbiased estimator for the population total.
3. Define MSD. Show that it is a valid probability sampling design.
4. Write all possible Modified Systematic Samples of size '8' from a population of size '40'.
5. Check whether or not Desraj ordered estimator is unbiased for the population total.
6. Explain the two types of Sampling Designs with suitable example for each type.
7. Find mean and variance of inclusion indicator under simple random sampling design.
8. Define the probability sampling design for Linear Systematic Sampling and explain its Sampling Scheme.
9. Verify if $\hat{Y}_{LR}$ is more efficient than $\hat{Y}_{R}$ unless $\beta = R$.
10. Discuss about Multistage Sampling Method.

SECTION – B

Answer any FIVE questions. Each carries EIGHT marks. (5 x 8 = 40 marks)

11. Explain Lahiri's method of sampling and prove that it is a PPS method.
12. In CSS, when the population is linear, prove that the usual expansion estimator is unbiased for the population total.
13. Discuss Simmons' unrelated randomized response model and find $\hat{\pi}_A$ when π_Y is unknown.
14. Under Cost Optimum Allocation, obtain the formula for 'n_h'.
15. For any fixed size sampling design, prove that

    (i) $\sum_{j=1}^{N} \pi_{ij} = (n - 1)\,\pi_i$; $j \neq i$, and

    (ii) $\sum_{j=1}^{N} (\pi_i \pi_j - \pi_{ij}) = \pi_i (1 - \pi_i)$; $i = 1,2, \ldots, N$; $j \neq i$.

16. For any sample size "n", find the estimated variance of $\hat{Y}_{DR}$.
17. For all samples receiving positive probabilities under Midzuno Sampling Design, show that the estimated variance of $\hat{Y}_{HT}$ is non-negative.
18. Derive Yates's corrected estimator for estimating population total without error, under LSS, when the population is linear.

SECTION – C

Answer any TWO questions. Each carries TWENTY marks. **(2 x 20 = 40 marks)**

19. (a) Illustrate that one can have more than one unbiased estimator for a given sampling design. (10)

(b) Find Π_i and Π_{ij} under SRSWOR, and hence deduce $\hat{Y}_{HT}$ and V ($\hat{Y}_{HT}$). (10)

20. (a) Write a detailed note on the Hartley-Ross unbiased ratio type estimator. (10)

(b) Explain the procedure of obtaining linear regression estimator for population total. (10)

21. (a) Discuss the procedure of 'Ratio Estimation' and obtain the approximate bias and mean square error of the estimator of 'population total'. (10)

(b) Find $\hat{Y}_{St}$, V ($\hat{Y}_{St}$) and v ($\hat{Y}_{St}$) under 'SRSWOR' and 'PPSWR'. (10)

22. (a) A SRS of size $n = n_1 + n_2$ with mean $\frac{\wedge}{Y}$ is drawn from a finite population of N units and a SR Subsample of size n_1 is drawn from it with mean $\frac{\wedge}{Y_1}$. Derive V ($\frac{\wedge}{Y_1}$ - $\frac{\wedge}{Y_2}$), where $\frac{\wedge}{Y_2}$ is the mean of the remaining n_2 units in the sample. (12)

(b) Obtain the variance of

(i) Hansen – Hurwitz estimator in double sampling.

(ii) Estimator $\hat{Y}_{TS}$ in Two – Stage Sampling. (8)

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